# Density functional theory and Møller-Plesset studies of hindered rotations of acetone 

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#### Abstract

The hindered rotations of acetone were studied density functional theory (B3LYP) and second order Møller-Plesset approaches using 6-31G** and $6-311 \mathrm{G}^{* *}$ basis sets. One of the $\mathrm{CH}_{3}$ groups of acetone with fixed heavy atoms was rotated from 0.0 to $120^{\circ}$, and CCH angles were scanned from 90.3 to $130.3^{\circ}$ to cover the potential energy surface of interest; a circular valley was obtained with the deepest potential value at a CCH angle equal to $109.3^{\circ}$. Potential energy profiles were then calculated by assuming that the molecular geometry could relax during rotation (i.e., each value of the torsion angle of the molecular geometry was optimized). Next, the two methyl groups were both rotated clockwise, and then one was rotated clockwise and the other counterclockwise. Using the variation method, and utilizing the first 20 harmonic oscillator wave functions, the energy levels, relative transition moment and relative transition intensities of the component of the hindered rotation $\nu_{2}\left(125.16 \mathrm{~cm}^{-1}\right)$ were computed in a onedimensional Schrodinger equation. The first three energy levels were almost degenerate; the next three were opened up, and the seventh energy level appeared above the level where tunneling can occur.


Keywords Hindered rotation • Potential energy profiles •
Potential energy surface • Torsion oscillation •
Variation method infrared transitions

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## Introduction

Acetone is used primarily as a chemical intermediate and as a solvent in both chemical and pharmaceutical applications. It is also very important in industrial and chemical reactions, being used as a raw material for the chemical synthesis of a wide range of products. The optimized structure of acetone with atomic numbering is given in Fig. 1.

Acetone is the molecular prototype for the aliphatic ketone series of molecules. In this molecule, the two methyl groups attached to the carbonyl are quite flexible, and are able to undergo torsional oscillation. Barriers that restrict such movement are known to be relatively low in this molecule, and thus the two $\mathrm{CH}_{3}$ groups undergo internal rotation. As the rotors are attached to the same carbon atom, the hydrogen atoms of the methyl groups come into proximity, and can sterically interfere, with each other. The low frequency dynamics of coupled methyl rotors have been the subject of extensive review. As the torsion motions extend over large amplitudes, it is necessary to consider the complete potential energy surface (PES) defined by $360^{\circ}$ rotations of the two torsion angles [1-3].

The microwave spectrum of acetone was first studied by Swalen and Costain [4] and later by Peters and Dreizler [5]. From the rotational spectrum patterns, these authors were able to determine an effective barrier for internal rotation. Vacherand et al. [6] carried out a complete analysis over a wider spectral range and refined the effective barrier, $V_{\text {eff }}=$ $266.09 \mathrm{~cm}^{-1}$. The most detailed study of the spectrum was that of Groner et al. [7]. These authors were able to observe the splitting of the bands into their torsional components at 125.16, 102.92, and $92.9 \mathrm{~cm}^{-1}$ in the infrared active $\mathrm{v}_{17}\left(b_{1}\right)$ band while considering $\mathrm{C}_{2 \mathrm{v}}$ symmetry.

The aim of this paper was to study (1) the complete PES, where the $\mathrm{CH}_{3}$ group is rotated internally by $360^{\circ}$ and

Fig. 1 Second order Møller-
Plesset (MP2)/6-311G**
optimized structure of acetone

makes an umbrella motion $(\mathrm{C}-\mathrm{C}-\mathrm{H}$ angle change from $90.3^{\circ}$ to $130.0^{\circ}$ ); (2) semi-rigid (all heavy atoms fixed, and only hydrogen atoms rotate at the most stable fixed angle of $\mathrm{C}-\mathrm{C}-\mathrm{H}, 109.3^{\circ}$ ) potential energy profiles (PEP); and (3) the most reliable fully relaxed (only the $\mathrm{O}-\mathrm{C}-\mathrm{C}-\mathrm{H}$ dihedral angle is fixed and all bond lengths and bond angles are optimized) PEP. Having wells below the saddle points, we will be able, through Hamiltonian construction and using the one-dimensional Schrodinger equation and variation method, to calculate energy levels and wave functions. The energy values are used to predict the frequencies, and the wave functions to calculate transition moments and relative intensities of far infrared transitions. The splitting of the levels depends on the degree of restriction of the rotational motion. Thus, at the bottom of the well, levels split by torsion may be close together and degenerate, whereas at the top of the well, they will be widely separated. Three transitions are expected to approximate experimental values.

## Computational details

Gaussian 03 W v.6.0 [7] was the computer program used, and all calculations were carried out at density functional theory (DFT) [8] and second order MøllerPlesset (MP2) [9] levels. The underlying DFT theory was the Khon and Sham approach to DFT, which uses a one particle Schrodinger equation and was performed using self consistent field (SCF) procedures. The Vosko, Wilk and Nusair (VWN) formula [10] was used for local density approximation for general gradient approximation (GGA) of the combination of Becke's three-parameter adiabatic connection exchange functional [11] with Lee-Yang-Parr correlation [12]; a list of B3LYP components was selected. The Møller-Plesset [13] approach was a second order perturbation theory. In both approaches, the large standard basis sets of $6-31 \mathrm{G}^{* *}$ and $6-311 \mathrm{G}^{* *}$ were used.

## Results

## Structural

First, the full optimization of acetone was carried out at the MP2 and B3LYP/31G** calculation levels without applying a symmetry option. The calculations showed that the molecule has $\mathrm{C}_{2 \mathrm{v}}$ symmetry (Table 1). One hydrogen atom of each methyl group, H 5 and H 8 , lies in the $\mathrm{O}-\mathrm{C}-\mathrm{C}$ plane and eclipses the oxygen. In a $120^{\circ} \mathrm{CH}_{3}$ rotation, the $\mathrm{C}-\mathrm{H}$ bond length and $\mathrm{C}-\mathrm{C}-\mathrm{H}$ bond angle change to $0.0046 \AA$ and $0.3376^{\circ}$, respectively, and the $\mathrm{CH}_{3}$ internal rotation axis forms an angle of approximately $0.17^{\circ}$ with the $\mathrm{C}-\mathrm{C}$ bond. These changes are small, and all $\mathrm{C}-\mathrm{H}$ bond lengths and all $\mathrm{C}-\mathrm{C}-\mathrm{H}$ and $\mathrm{H}-\mathrm{C}-\mathrm{H}$ bond angles could be considered the same. In this case, the $\mathrm{CH}_{3}$ rotational axis lies along the $\mathrm{C}-\mathrm{C}$ bond. With the new settings, the B3LYP/ $6-31 \mathrm{G}^{* *}$ calculation showed $\mathrm{C}-\mathrm{C}-\mathrm{H}=110.23^{\circ}$, $\mathrm{H}-\mathrm{C}-\mathrm{H}=$ $108.70^{\circ}, \mathrm{C}-\mathrm{O}=1.2158$ Á, $\mathrm{C}-\mathrm{H}=1.0943$ Á and $\mathrm{C}-\mathrm{C}=$ $1.5201 \AA$, which were in better agreement with experimental values [14].

Next, optimizations were performed at MP2 and B3LYP/ $6-311 \mathrm{G}^{* *}$ levels of theory. The molecule turned out to optimize with $\mathrm{C}_{1}$ symmetry; the hydrogen atom is no longer in the $\mathrm{O}-\mathrm{C}-\mathrm{C}$ plane, and the $\mathrm{O}-\mathrm{C}-\mathrm{C}-\mathrm{H}$ dihedral angles are $+8.5^{\circ}$ clockwise and $-8.5^{\circ}$ counterclockwise of the methyl groups. A small barrier with a height of $3.8 \mathrm{~cm}^{-1}$ at the $\mathrm{O}-$ $\mathrm{C}-\mathrm{C}-\mathrm{H}$ dihedral angle of $\pm 0^{\circ}$ appeared (Fig. 1). Thus, the experimental structural and vibration analysis based on $\mathrm{C}_{2 \mathrm{~V}}$ symmetry considerations needs to be revised.

Potential energy surface

The PES for one $\mathrm{CH}_{3}$ rotation was constructed by approximating the $\mathrm{C}-\mathrm{H}$ bond length and $\mathrm{C}-\mathrm{C}-\mathrm{H}$ bond angles to be equal. The dihedral angles of $\mathrm{H}-\mathrm{C}-\mathrm{C}-\mathrm{O}, \theta_{1}$, from 0.0 to $120.0^{\circ}$, and $\mathrm{C}-\mathrm{C}-\mathrm{H}$ angles, $\varphi$, from 90.3 to $130.3^{\circ}$, were scanned at $1^{\circ}$ steps; 4,961 single point calculations were completed. The constructed PES was

Table 1 Second order Møller-Plesset (MP2)- and B3LYP-optimized bond lengths and bond angles

|  | MP2/6-311G** |  | B3LYP/6-311G** |  | Experimental [10] |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{C}_{2} \mathrm{~V}$ | $\mathrm{C}_{1}$ | $\mathrm{C}_{2} \mathrm{~V}$ | $\mathrm{C}_{1}$ |  |
| Bond length (Á) |  |  |  |  |  |
| C1-O2 | 1.2167 | 1.2165 | 1.2074 | 1.2091 | 1.215 |
| C1-C3 | 1.5176 | 1.5179 | 1.5179 | 1.5191 | 1.515 |
| C1-C4 | 1.5176 | 1.5175 | 1.5179 | 1.5191 | 1.515 |
| C3-H7 | 1.0901 | 1.0901 | 1.0868 | 1.0890 | 1.086 |
| C3-H8 | 1.0947 | 1.0941 | 1.0923 | 1.0943 | 1.086 |
| C3-H9 | 1.0947 | 1.0995 | 1.0923 | 1.0950 | 1.086 |
| C4-H10 | 1.0901 | 1.0901 | 1.0868 | 1.0890 | 1.086 |
| C4-H11 | 1.0947 | 1.0941 | 1.0923 | 1.0943 | 1.086 |
| C4-H12 | 1.0947 | 1.0995 | 1.0923 | 1.0950 | 1.086 |
| Bond angle ( ${ }^{\circ}$ ) |  |  |  |  |  |
| O2-C1-C3 | 122.002 | 122.022 | 121.768 | 121.772 | 121.800 |
| O2-C1-C4 | 122.002 | 122.022 | 121.768 | 121.772 | 121.800 |
| C3-C1-C4 | 115.997 | 115.956 | 116.463 | 116.457 | 116.400 |
| C1-C3-H7 | 109.665 | 109.345 | 109.673 | 109.695 | 110.267 |
| C1-C3-H8 | 110.003 | 110.591 | 110.417 | 110.835 | 110.267 |
| C1-C3-H9 | 110.003 | 109.721 | 110.417 | 109.963 | 110.267 |
| C1-C3-H8 | 109.665 | 109.345 | 109.673 | 109.695 | 110.267 |
| C1-C3-H9 | 110.003 | 110.591 | 110.417 | 110.835 | 110.267 |
| C1-C3-H10 | 110.003 | 109.721 | 110.417 | 109.963 | 110.267 |
| $\mathrm{O} 2-\mathrm{C} 1-\mathrm{C} 3-\mathrm{H} 5$ | 0.0 | 8.5 | 0.0 | 6.0 | 0.0 |

projected on a flat equatorial plane; a net presentation and contour map are shown in Figs. 2 and 3, respectively. The circular valley in Fig. 2 (only one-third of the valley is presented) can be divided into two parts: the space over the saddle points (the most unstable configuration of acetone $\theta_{1}=45^{\circ}$ having the highest structural energy), and the wells below the saddle points. In the space over the saddle points, the internal methyl group can rotate freely. Below the saddle points, where three wells are located, out-of-plane bending could occur (hindered rotation or torsion oscillations of the $\mathrm{a}_{2}$ and $\mathrm{b}_{1}$ type). $\mathrm{C}-\mathrm{C}-\mathrm{H}$ in-plane bending (umbrella bending) uses both spaces. The slice of potential energy in the radial direction crossing through the bottom of the well, which is in the Morse-type potential, is the potential energy of the one $\mathrm{C}-\mathrm{C}-\mathrm{H}$ in-plane bend. The motion could be considered as one-dimensional, and its potential energy could be introduced into the Schrödinger equation. At points below the saddle, torsion motions of methyl groups occurred in three wells. In the torsion oscillation, all three hydrogen atoms were moving clockwise and counterclockwise from their equilibrium position, in harmony. Of course, two methyl groups could be coupled to each other as clockwiseclockwise and clockwise-counterclockwise.

## Semi-relaxed PEP

The steepest curve in the valley occurred at $\varphi=109.3^{\circ}$. Three circular PEPs were constructed at constant $\varphi$ and
with fixed heavy atoms ( O and C atoms). First, one of the methyl groups was rotated; the symmetrical barrier heights had values of MP2/6-311G** $=287.9 \mathrm{~cm}^{-1}$ and B3LYP/6-311G** $=277.6 \mathrm{~cm}^{-1}$. Second, two methyl groups were rotated, both clockwise. The corresponding heights were $1,061.2 \mathrm{~cm}^{-1}$ and $941.3 \mathrm{~cm}^{-1}$ for MP2 and B3LYP, respectively. Third, one methyl group was rotated clockwise and the other counterclockwise; the barrier heights were $1,058.2 \mathrm{~cm}^{-1}$ and $941.3 \mathrm{~cm}^{-1}$ using MP2 and B3LYP with the same basis sets, respectively. The straightened potential energy profile closely obeyed a sinusoidal function $V=V_{3}(1-\operatorname{Cos} 3 \alpha) / 2$, where $V_{3}$ is the internal rotation barrier, and the factor 3 was the internal rotation symmetry number. In order to compare calculated PEPs with the sinusoidal function, $V_{3}$ was set arbitrarily to $282 \mathrm{~cm}^{-1}$ for one methyl group rotation, and to $1,000 \mathrm{~cm}^{-1}$ for both methyl groups rotation (Fig. 4). The barrier height with both methyl groups rotated in the same direction was equal to one methyl group rotating clockwise and the other counterclockwise, but the width was narrower.

The barrier height of one methyl group rotation MP2/6$311 \mathrm{G}^{* *}$ ( $287.9 \mathrm{~cm}^{-1}$ ) was considerably less than one-half of the barrier height of the two methyl group rotations MP2/6-311G** (clockwise-clockwise; $1,061.2 \mathrm{~cm}^{-1}$ and clockwise-counterclockwise; $1,058.2 \mathrm{~cm}^{-1}$ ), which demonstrated that, if both rotors operate together, rotor coupling is important.


Fig. 2 Net representation of the projected potential energy surface (PES) on an equatorial plane. The circular valley represents the area of methyl internal motion. The cross-section shows the behavior of the
anharmonic potential function, the steeper part reflects a small centrifugal distortion
0.0 to 120 at the proposed levels of calculation. The ZMatrix was constructed in such a way that, by changing $\theta_{1}$ and $\theta_{2}$, other $\mathrm{O}-\mathrm{C}-\mathrm{C}-\mathrm{H}$ dihedral angles change with the same $2^{\circ}$ steps. In one methyl rotation, only $\theta_{1}$, and in two methyl rotations, $\theta_{1}$ and $\theta_{2}$ were kept constant and all other structural variables were subjected to optimization (Fig. 5).

Fig. 3 Contour map, showing clearly the location of one well. The area outside the scan area is generated by the Surfur computer program and has no meaning



Fig. 4 a One methyl rotation, $\mathbf{b}$ two methyl rotation (both clockwise), $\mathbf{c}$ two methyl rotation (one clockwise and the other counterclockwise)

The PEP wells were wider while the barriers were narrower than the corresponding cosine potential, which provided relatively easier tunneling at the seventh energy level.

The fully relaxed model represented a more reliable approach to the torsion potential surface problem. An important outcome of this model was significant lowering of the torsion potential barrier to $233.0 \mathrm{~cm}^{-1}$ (B3LYP/6$31 \mathrm{G}^{* *}$ ) and $265.6 \mathrm{~cm}^{-1}$, (MP2/6-31G**), from the microwave $270 \mathrm{~cm}^{-1}$ value (Table 2). The hydrogen atom with $(1 \mathrm{~s})^{1}$ electronic structure was a simple atom, and in calculations it may be more convenient to use $6-31 \mathrm{G}^{* *}$

## The most relaxed one methyl rotation



Fig. 5 The most relaxed one-methyl rotation, using MP2/6-311G**, V3 and B3LYP/6-311G** levels of calculations
rather than more sophisticated basis functions such as 6$311 \mathrm{G}^{* *}$. However, the second basis functions yield more detailed structure, and thus are more recommended.

An interesting point found using the fully relaxed model was that a small barrier occurred at $\theta_{1}=0.0^{\circ}$, whereas the most stable configurations (two configurations) appeared at $\theta_{1}= \pm 6.0^{\circ}, \theta_{2}=\mp 6.0^{\circ}$ and $\theta_{1}= \pm 8.5^{\circ}, \theta_{2}=\mp 8.5^{\circ}$ for B3LYP and MP2, respectively. The could be because the angle of rotation was fixed and all other variables were optimized, which was found to be the most preferred optimization.

## Vibration

The fully optimized $\mathrm{C}_{2 \mathrm{v}}$ and $\mathrm{C}_{1}$ structures at the most stable configuration, $\left(\theta_{1}=0.0^{\circ}\right.$ and $\theta_{2}=0.0^{\circ}$ for both B3LYP and MP2) and $\left(\theta_{1}= \pm 6.0^{\circ} \theta_{2}=\mp 6.0^{\circ}\right.$ for B3LYP, and $\theta_{1}= \pm 8.5^{\circ}$ $\theta_{2}=\mp 8.5^{\circ}$ for MP2), were subjected to harmonic and anharmonic vibration frequency calculations using 6-311G** basis sets (Table 3). Apart from very low frequencies, MP2 values were closer to experimental values and, in the high region, B3LYP frequencies were a better match with the corresponding experimentally determined frequencies [15, 16]. Inspection of Table 3 reveals that the differences between harmonic and corresponding anharmonic frequencies are not a fraction number. A scaling factor could be used to convert harmonic to anharmonic frequencies, but more precautions may be needed in this case.

The $\mathrm{CH}_{3}$ torsion-the vibration mode at $125 \mathrm{~cm}^{-1}$ —will be explored in depth because of its interesting triple split (125.2, 102.9, $92.9 \mathrm{~cm}^{-1}$ ). We will take advantage of the

Table 2 Barrier heights computed with different structural settings and levels of theory (all in $\mathrm{cm}^{-1}$ )

| Barrier heights, $\mathrm{CH}_{3}$ rotation | MP2 |  | B3LYP |  | Experimental |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 6-311G** | 6-31G** | 6-311G** | 6-31G** |  |
| Fully relaxed |  |  |  |  |  |
| One $\mathrm{CH}_{3}$ rotation | 224.6 | 265.6 | 212.8 | 233.0 | 270 |
| Both $\mathrm{CH}_{3}$ clockwise rotation | 797.0 | 784.8 | 705.0 | 724.0 |  |
| $\mathrm{CH}_{3}$ clockwise and counterclockwise rotation | 793.2 | 787.1 | 704.4 | 727.0 |  |
| Semi-relaxed |  |  |  |  |  |
| One $\mathrm{CH}_{3}$ rotation | 287.9 | 322.5 | 277.6 | 291.7 | 270 |
| Both $\mathrm{CH}_{3}$ clockwise rotation | 1,061.2 | 1,039.5 | 941.3 | 946.5 |  |
| $\mathrm{CH}_{3}$ clockwise and counterclockwise rotation | 1,058.2 | 1,039.5 | 941.3 | 946.5 |  |

variation method to solve the Schrodinger vibration equation of the system.

The variation method
If a molecule can assume more than one stable configura-tion-in this case stable implies that the configuration is associated with the bottom of a potential well-we would have multiple well potentials. Such configurations are often equivalent and can be defined conveniently with a cosine
function for each respective well. In the present case, the situation was more complicated. The internal rotation of methyl groups was previously thought to have three stable configurations, but our findings revealed that the molecule did not have a $\mathrm{C}_{2 \mathrm{v}}$ symmetry. Methyl groups in their stable configurations were rotated clockwise-counterclockwise about $8.5^{\circ}$ leaving two stable conformations. Altogether six minimum PEPs $\left(360^{\circ}\right)$ were detected.

The dynamics, such as stretching, of a microscopic particle can be described with the one-dimensional Schrö-

Table 3 Vibrational band assignment of acetone (frequencies in $\mathrm{cm}^{-1}$ )

| No. | Assignment | B3LYP/6-311G** $\mathrm{C}_{2} \mathrm{~V}$ |  | B3LYP/6-311G** $\mathrm{C}_{1}$ |  | MP2/6-311G** $\mathrm{C}_{2} \mathrm{~V}$ |  | MP2/6-311G** $\mathrm{C}_{1}$ |  | Experimental$[11,12]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Harmonic | Anharmonic | Harmonic | Anharmonic | Harmonic | Anharmonic | Harmonic | Anharmonic |  |
| 1 | $\mathrm{CH}_{3}$ torsion, $\mathrm{a}_{2}$ | -60.2 | -48.7 | 56.2 | -1056.1 | -86.5 | -164.8 | 79.3 | -205.7 | 77 |
| 2 | $\mathrm{CH}_{3}$ torsion, $\mathrm{b}_{1}$ | 114.8 | 139.8 | 139.0 | 8.8 | 127.2 | 117.2 | 139.0 | 139.6 | 125 |
| 3 | C-C-C bend, $\mathrm{a}_{1}$ | 373.0 | 390.1 | 377.3 | 380.6 | 375.2 | 378.3 | 376.7 | 385 | 385 |
| 4 | $\mathrm{CH}_{2}$ twist, $\mathrm{b}_{1}$ | 491.8 | 494.0 | 488.9 | 476.8 | 488.4 | 486.2 | 481.4 | 485.2 | 484 |
| 5 | Skeleton, $\mathrm{b}_{2}$ | 528.3 | 528.6 | 535.6 | 530.0 | 533.6 | 529.5 | 534.4 | 531.5 | 530 |
| 6 | C-Csym. str., $\mathrm{a}_{1}$ | 783.7 | 767.7 | 779.2 | 766.4 | 804.0 | 787.2 | 803.5 | 789.3 | 777 |
| 7 | $\mathrm{CH}_{3}$ bend, $\mathrm{a}_{2}$ | 885.7 | 884.8 | 882.6 | 856.8 | 889.5 | 882.8 | 889.1 | 861.2 | 877 |
| 8 | $\mathrm{CH}_{2}$ twist, $\mathrm{b}_{2}$ | 892.8 | 877.1 | 884.0 | 826.6 | 898.4 | 885.8 | 906.1 | 877.1 | 891 |
| 9 | $\mathrm{CH}_{3}$ bend, $\mathrm{a}_{1}$ | 1,082.1 | 1,068.8 | 1,083.1 | 1,051.8 | 1,087.6 | 1,065.0 | 1,091.3 | 1,070.5 | 1,066 |
| 10 | $\mathrm{CH}_{2}$ twist, $\mathrm{b}_{1}$ | 1,125.8 | 1,099.2 | 1,117.3 | 1,095.9 | 1,127.3 | 1,100.0 | 1,121.2 | 1,101.4 | 1,091 |
| 11 | C-C str. , $\mathrm{b}_{2}$ | 1,234.7 | 1,202.3 | 1,230.3 | 1,194.8 | 1,252.1 | 1,217.8 | 1,253.9 | 1,220.9 | 1,216 |
| 12 | $\mathrm{CH}_{3}$ umbrella bend, $\mathrm{b}_{2}$ | 1,394.5 | 1,365.3 | 1,383.0 | 1,347.7 | 1,395.5 | 1,361.3 | 1,395.4 | 1,360.9 | 1,364 |
| 13 | $\mathrm{CH}_{3}$ umbrella bend, $\mathrm{a}_{1}$ | 1,397.5 | 1,368.6 | 1,385.5 | 1,343.5 | 1,404.1 | 1,368.7 | 1,405.8 | 1,368.6 | 1,364 |
| 14 | CH2 wagg., $\mathrm{b}_{2}$ | 1,466.6 | 1,437.1 | 1,461.2 | 1,405.1 | 1,476.7 | 1,438.6 | 1,482.3 | 1,441.2 | 1,410 |
| 15 | CH2 twist, $\mathrm{a}_{2}$ | 1,475.6 | 1,445.9 | 1,466.3 | 1,425.2 | 1,485.8 | 1,449.5 | 1,486.2 | 1,448.7 | 1,426 |
| 16 | CH2 wagg., $\mathrm{a}_{1}$ | 1,488.1 | 1,453.5 | 1,471.2 | 1,422.7 | 1,492.4 | 1,453.6 | 1,491.6 | 1,452.1 | 1,435 |
| 17 | CH2 twist, $\mathrm{b}_{1}$ | 1,509.6 | 1,477.2 | 1,488.2 | 1,433.1 | 1,513.0 | 1,472.1 | 1,506.8 | 1,461.7 | 1,454 |
| 18 | $\mathrm{C}=\mathrm{O}$ str. , $\mathrm{a}_{1}$ | 1,822.9 | 1,797.0 | 1,804.2 | 1,780.9 | 1,787.4 | 1,728.2 | 1,787.7 | 1,740.0 | 1,731 |
| 19 | $\mathrm{CH}_{3}$ sym. str. , $\mathrm{a}_{1}$ | 3,047.1 | 2,947.9 | 3,024.5 | 2,908.8 | 3,079.0 | 2,977.8 | 3,075.3 | 2,974.1 | 2,937 |
| 20 | $\mathrm{CH}_{3}$ sym. str., $\mathrm{b}_{2}$ | 3,053.0 | 2,954.0 | 3,031.3 | 2,918.3 | 3,083.4 | 2,981.5 | 3,080.0 | 2,979.2 | 2,937 |
| 21 | $\mathrm{CH}_{2}$ sym. str. , $\mathrm{a}_{2}$ | 3,125.5 | 2,985.5 | 3,080.1 | 2,922.9 | 3,177.7 | 3,038.8 | 3,158.2 | 3,020.2 | 2,963 |
| 22 | $\mathrm{CH}_{2}$ asym. str. , $\mathrm{b}_{1}$ | 3,132.5 | 2,989.1 | 3,087.1 | 2,936.0 | 3,182.9 | 3,042.1 | 3,163.1 | 3,027.2 | 2,972 |
| 23 | $\mathrm{CH}_{3}$ asym. str. , $\mathrm{a}_{1}$ | 3,136.5 | 2,990.4 | 3,139.8 | 2,987.7 | 3,183.4 | 3,042.7 | 3,204.3 | 3,069.6 | 3,019 |
| 24 | $\mathrm{CH}_{3}$ asym. str. , $\mathrm{b}_{2}$ | 3,138.1 | 2,992.0 | 3,141.0 | 2,988.4 | 2,992.0 | 3,043.4 | 3,205.4 | 3,067.6 | 3,019 |

dinger equation. However, the potential energy function was complicated. Next, we attempted the variation method for the treatment of the problem, i.e., to treat the problem in one dimension, the circular hindered internal rotation potential was opened up and straightened.

Introducing the non-dimensional variables $\xi$ and $\lambda$ as [19]:
$\xi=\left(\frac{\mu \omega_{e}}{h}\right)^{2} R \quad \lambda=\frac{2 E}{h \omega_{e}}$
where $\hbar$ is Plank's constant divided by $2 \pi, R$ is the variation in inter-nuclear distance, $\mu$ is the reduced mass of the involved nuclei, and $\omega_{\mathrm{e}}$ is the arbitrary angular vibration frequency. In the $\xi$ scale, the Schrodinger equation takes the following form:
$\frac{d^{2} \Psi(\xi)}{d \xi^{2}}-\frac{2 V(R)}{\hbar \omega_{e}} \Psi(\xi)=-\lambda \Psi(\xi)$
where $V(R)$ is the potential energy of the torsion motion. Due to the lack of mathematical potential energy, a direct solution of Eq. 2 is not possible; therefore, the variation method will be applied. The wave function is expanded as:
$\Psi(\xi)=\sum_{i=0}^{n} C_{i} \Phi_{i}(\xi)$
where $\Phi_{\mathrm{i}}(\xi)$ is simple harmonic oscillator wave function. The term $H_{\mathrm{jk}}$ is defined as:
$\mathrm{H}_{j k}=\mathrm{HK}_{j k}+\mathrm{H} U_{j k}$
where
$\mathrm{H} K_{j k}=\int \Phi_{j}^{*}(\xi) \frac{d^{2}}{d \xi^{2}} \Phi_{k}(\xi) d \xi$
$\mathrm{H} U_{j k}=\int \Phi_{j}^{*}(\xi) \frac{2 \mu V(R)}{h \omega_{e}} \Phi_{k}(\xi) d \xi$
Nonzero matrix elements of the $H K_{\mathrm{jk}}$, are as follows:

| $J$ | $k$ | $H K_{j k}$ |
| :--- | :--- | :--- |
| $J$ | $j-2$ | $[j(j-4) / 4]^{1 / 2}$ |
| $J$ | $j$ | $(j+1 / 2)-(2 j+1)$ |
| $J$ | $j+2$ | $[(j+2)(j+1) / 4]^{1 / 2}$ |

The $H U_{\mathrm{jk}}$ integrals were calculated numerically using the trapezoid method. The n in Eq. 3 was taken as 19 and the generated $\mathrm{H}_{\mathrm{ij}}$ matrix was diagonalized for eigenvalues. Eigenfuctions were extracted from unitary transforming matrices.

The calculated relative transition moments, $\mathrm{R}_{\mathrm{ij}}$, relative transition probability, $\mathrm{A}_{\mathrm{ij}}$, relative transition intensity, $\mathrm{I}_{\mathrm{ij}}$ and transition frequency, $\nu_{\mathrm{ij}}$, are listed in Table 4 [15]. According to Table 4, more than three transitions were expected. These may be detected with very high resolution instruments. Using a Lorenzian line shape with a $2-$ $10 \mathrm{~cm}^{-1}$ full line width, three observed components in the form of IR absorption spectra were shown (Fig. 6). In Fig. 6, calculated IR intensities were utilized to construct the spectrum. The low intensity of the component at $92.9 \mathrm{~cm}^{-1}$ was probably the reason for its late experimental detection. Six energy levels were located in the hindered rotation area. First, three energy states were thought to be degenerate, but two stable conformations caused further splitting and, in a few, wave numbers energy levels repelled each other. However, energy levels 4-6 were opened up

Table 4 Components of the $v_{2}$ torsional frequency (in $\mathrm{cm}^{-1}$ )

| I | j | $\nu_{\mathrm{ij}}$ | $\mathrm{R}_{\mathrm{ij}}$ | $\mathrm{A}_{\mathrm{ij}}$ | $\mathrm{I}_{\mathrm{ij}}$ | $\nu_{\mathrm{ij},}$, Experimental [17] | $\gamma_{\mathrm{ij}}$, Experimental [18] |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 2 | 3.6 | 0.6025 | 0.0251 | 0.0251 |  |  |
| 1 | 3 | 6.6 | -0.0428 | 0.0002 | 0.0002 |  |  |
| 1 | 4 | 100.8 | -0.6602 | 0.8369 | 0.8369 | 104.5 |  |
| 1 | 5 | 127.5 | 0.2010 | 0.0981 | 0.0981 |  |  |
| 1 | 6 | 133.7 | 0.0787 | 0.0158 | 0.0158 |  |  |
| 1 | 7 | 203.3 | 0.0060 | 0.0001 | 0.0001 |  |  |
| 2 | 3 | 3.0 | 4.1830 | 0.9940 | 0.9768 |  |  |
| 2 | 4 | 97.2 | -0.0986 | 0.0180 | 0.0177 |  |  |
| 2 | 5 | 123.8 | 0.4354 | 0.4470 | 0.4393 | 124.5 | 124.5 |
| 2 | 6 | 130.1 | -0.6163 | 0.9414 | 0.9251 |  |  |
| 2 | 7 | 199.7 | 0.2023 | 0.1557 | 0.1530 |  |  |
| 3 | 4 | 94.2 | -0.3035 | 0.1653 | 0.1601 |  |  |
| 3 | 5 | 120.8 | 0.2159 | 0.1073 | 0.1039 | 125.2 |  |
| 3 | 6 | 127.1 | 0.3101 | 0.2330 | 0.2256 | 124.5 |  |
| 3 | 7 | 196.7 | -0.0535 | 0.0107 | 0.0104 |  |  |

The components of the torsional frequency at 125 cm-1


Fig. 6 The components of the torsion frequency at $125 \mathrm{~cm}^{-1}$
widely and many transitions occurred among the energy levels. The seventh energy level was located above the mouth of the barrier height. This indicated that if the quantum mechanical tunneling effect happened at all, it was going to be in the 5th and 6th states (Figs. 7, 8).


Fig. 7 PEP energy levels, first and sixth wave functions

PEP, Energy levels and Probabilities


Fig. 8 PEP energy levels, first and sixth probabilities (square of the wave functions)

## Conclusion

The fully relaxed model represents a more reliable physical approach to the torsion potential surface. An important outcome of this model was the significant lowering of the torsion potential barrier to $224.6 \mathrm{~cm}^{-1}$ from the microwave $270 \mathrm{~cm}^{-1}$ value. The utility of Rydberg spectroscopy in developing a torsion barrier potential model has been demonstrated. Secure ground-state $\nu_{12}$ torsion vibration frequency information allowed clear discrimination between the rigid-frame and fully relaxed models and construction of a unique ground-state empirical torsion potential.

Concerning structural properties, it was found that the length of six $\mathrm{C}-\mathrm{H}$ bonds was not equal, and two methyl groups were rotated clockwise and counterclockwise about $8.5^{\circ}$. Potential energy and frequencies were very sensitive to the molecular geometry, hence equilibrium structures must be chosen carefully. Large and different scaling factors (often $5 \%-10 \%$ ) were needed in harmonic oscillator consideration in different areas of the IR spectrum. In summary, we obtained the best result from an anharmonic oscillator with MP2, B3LYP with basis set 6-31G, 6-311G**.

Acetone, with its $\mathrm{C}_{2} \mathrm{v}$ rigid frame and its $\mathrm{C}_{3} \mathrm{v}$ methyl rotors, may be regarded as a prototype system for studying the effects of strongly coupled large amplitude motion. For this purpose, the torsion frequencies were calculated from sophisticated DFT and second order perturbation theory calculations. Correlation between the calculated and experimental spectra has allowed assignment of the major bands using the Gauss view computer program.

In this paper, the multiple structures that result from the transitions between torsion microstates have been well reproduced when compared with the available experimental data. The frequency calculated with the MP2/6-311G**
with $\mathrm{C}_{1}$ symmetry, $139.9 \mathrm{~cm}^{-1}$, was found to be slightly higher than the experimental value of $125.16 \mathrm{~cm}^{-1}$. Thus, the barrier height and complex form of the PEP, which controls the torsion frequency, was calculated to be slightly higher than the value from microwave findings.

The complex form of the PEP caused the first three energy levels of the torsion to repel each other about few wave numbers, but the next three energy levels were opened up widely. The calculated transition moment provided several transitions and generated IR absorption spectra with full band width of $2-10 \mathrm{~cm}^{-1}$. This showed three bands at a few $\mathrm{cm}^{-1}$ deviations from experimental values. The low intensity of the band at $93 \mathrm{~cm}^{-1}$ may be the reason for its experimentally late observation. In the future, very high resolution spectrophotometers may be able to detect other bands.

A likely explanation for the differences between the calculated and the observed frequencies could be the assumptions made in the calculations. Apart from the assumptions made in calculation, the model assumed that, while the torsion modes couple strongly with each other, they did not interact with the other modes in the molecule. The $\mathrm{C}-\mathrm{C}-\mathrm{O}-\mathrm{C}$ molecular frame of acetone underwent skeletal vibrations. $\mathrm{C}-\mathrm{C}-\mathrm{C}$ bend at $385 \mathrm{~cm}^{-1}$ and $\mathrm{C}-\mathrm{H}_{2}$ twist at $484 \mathrm{~cm}^{-1}$ occurred at relatively low frequencies and could couple with the torsion modes. Interactions between either of these frame modes and the methyl tops would push and compress the torsion levels downwards and could thus account for the differences between the observed and calculated frequencies.

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